Persistence Probabilities of the German DAX and Shanghai Index

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Abstract

We present a relatively detailed analysis of the persistence probability distributions in financial dynamics. Compared with the auto-correlation function, the persistence probability distributions describe dynamic correlations non-local in time. Universal and non-universal behaviors of the German DAX and Shanghai Index are analyzed, and numerical simulations of some microscopic models are also performed. Around the fixed point $z_0 = 0$, the interacting herding model produces the scaling behavior of the real markets.

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1 Introduction

Recently, much attention of physicists has been drawn to dynamics of financial markets. From the view of many-body systems, interactions among agents and producers may generate long-range temporal correlations in financial dynamics, and therefore result in the so-called dynamic scaling behavior. To clearly observe the scaling behavior, one needs to carefully investigate the dynamics at the 'microscopic' level.

Fortunately, in the past years, it has been piled up large amount of data in financial markets, especially those records of economic indices in *minutes or seconds*. This allows a relatively accurate analysis of the financial dynamics at the 'microscopic' level. In 1995, Mantegna and Stanley had carefully analyzed the data of a stock market index in US — the Standard and Poor 500 [1]. The probability distribution $P(\Delta y, \Delta t)$ of the price change Δy in a time Δt obeys a dynamic scaling form.

Inspired by the work of Mantegna and Stanley [1], many activities have been devoted to the study of financial markets [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. More systematic investigation of the tails of $P(\Delta y, \Delta t)$ and the volatility of price fluctuations in the Standard and Poor 500 has been presented [13, 14]. In spite of *absence* of the two-point correlation of the variations, the volatility is long-range correlated. This is believed to be the physical origin of the universal scaling behavior in financial dynamics. On the other hand, different models and theoretical approaches have been also developed to describe financial markets [9, 12, 15, 16, 17, 18, 10, 19, 20, 21, 22, 23]. Among them, important examples are Minority games and percolation models and their variants.

Up to date, stationary properties of financial indices and the dynamic behavior local in time are mainly concerned. For full understanding of financial markets, however, the dynamic behavior *non-local* in time should be also very important in

theory and practice. An interesting example is the so-called persistence probability, which has been systematically investigated in non-equilibrium dynamics such as phase ordering dynamics and critical dynamics. As time evolves, the persistence probability decays by a power law characterized by a persistence exponent. It is shown that the persistence exponent is in general *independent of* other known critical exponents [24]. The persistence exponent plays an important role in a variety of systems and has been directly measured in experiments [25, 26, 27, 28, 29].

The persistence probability is not only important in the fundamental theory of the non-equilibrium dynamics, but also helpful in thorough understanding of financial markets. The persistence probability directly defined with the price of the index $y(t')$, is first introduced in financial dynamics [30]. The persistence exponent θ_p is estimated to be 0.49(2). It is very close to that of a random walk, even though the probability distribution $P(\Delta y, \Delta t)$ of the financial index is significantly different from Gaussian. This result should be related to the fact that $\Delta y(t') = y(t' + \Delta t) - y(t')$ is short-range correlated in time.

Consequently, the persistence probability defined with the *magnitude* of the variation $\Delta y(t')$ of the index is investigated [31]. Since the magnitude of the variation $\Delta y(t')$ is long-range correlated in time, one expects a non-trivial behavior of the persistence probability distribution. Some preliminary results of this kind have been reported for the German DAX in Ref. [31]. In this paper, a more detailed analysis on this topic will be presented, and further extended to the case away from the fixed point. In addition to the German DAX, the Shanghai Index in China is also carefully analyzed, and universal and non-universal behaviors of two economic indices are revealed. Although up to date not much work has been devoted to the Chinese stock market, it should be rather interesting to study the dynamic behavior of the Chinese market, in comparison with that of the financial market in western countries. This may help understand what is real universal in stock markets. Finally,

numerical simulations of an interacting herding model [32] and a Minority Game model [20] are briefly reported.

In Sec. 2, the persistence probability distribution is introduced and investigated. In Sec. 3, it is generalized to the case away from the fixed point. Finally, conclusions come in Sec. 4.

2 Persistence probabilities

2.1 The German DAX and Shanghai Index

Let us denote the value of a index at a certain time t' as $y(t')$, and the magnitude of the logarithm price change in a fixed time interval Δt as $Z(t', \Delta t) \equiv |\ln y(t' + \Delta t) - \ln y(t')|$. We define the persistence probability $P_+(t)$ ($P_-(t)$) as the probability that $Z(t' + \tilde{t}, \Delta t)$ has always been above (below) $Z(t', \Delta t)$ in time t, i.e., $Z(t' + \tilde{t}, \Delta t) > Z(t', \Delta t)$ ($Z(t' + \tilde{t}, \Delta t) < Z(t', \Delta t)$) for all $\tilde{t} < t$. The average is taken over the time variable t'.

Here $Z(t', \Delta t)$ is defined as the magnitude of the variation of $\ln y(t')$, slightly different from that in Ref. [31]. Taking $\ln y(t')$ in the definition of $Z(t', \Delta t)$, one may eliminate possible effects of the background evolution of the index. In Ref. [31], $Z(t', \Delta t)$ is defined as the magnitude of the variation of $y(t')$, and therefore, the background evolution should be directly removed in $y(t')$. Both definitions of $Z(t', \Delta t)$ yield similar results, but that with $\ln y(t')$ is slightly more fluctuating. In the present paper, we adopt the definition with $\ln y(t')$ since the total time interval of the Shanghai Index is too short for figuring out the the background evolution.

The persistence probabilities describe the temporal correlation of $Z(t', \Delta t)$ *non-local* in time, while the auto-correlation function describes the temporal correlation local in time. In general, the persistence probabilities provide additional information to the auto-correlation function, and are less fluctuating in the measurements. Therefore, they are interesting and important for deep understanding of the finan-

cial dynamics. For a random walk process, both $P_+(t)$ and $P_-(t)$ decay by a power law with a persistence exponent $\theta_p = 1$. Since $Z(t', \Delta t)$ in financial dynamics is long-range correlated in time, one expects a non-trivial dynamic behavior.

We first perform the measurements using the minute-to-minute data of the German Dax from December 1993 to July 1997. The total number of the records during this period of time is about 350 000. In the measurements, we take $\Delta t = 1$ minute. In Fig. 1, we plot the persistence probabilities $P_+(t)$ and $P_-(t)$ on a log-log scale. It is obvious that $P_-(t)$ obeys a power law up to four orders of magnitude, while $P_+(t)$ decays to zero rather fast. Compared with the auto-correlation function calculated with the same data, $P_+(t)$ and $P_-(t)$ are much less fluctuating. Different behaviors of $P_+(t)$ and $P_-(t)$ indicate the high-low asymmetry in the time series of $y(t')$. For $P_-(t)$, we assume a power law

$$P_-(t) \sim t^{-\theta_p}, \tag{1}$$

and θ_p is the so-called persistence exponent.

Carefully looking at the curve of $P_-(t)$ of the German DAX, we observe a quasi-periodic dropping in the first 2000 minutes. The period is roughly one working day, i.e., about 350 to 480 minutes in those years. This behavior should be traced back to the disconnection of the index between two successive days. $Z(t', \Delta t = 1\ min)$ at the last minute of a day is much bigger than those during the day. This disconnection affects the behavior of $P_-(t)$ in the first periods of time. When one measures the slope in a time interval $[500, 20000]$, the persistence exponent of $P_-(t)$ is $\theta_p = 0.88(2)$, clearly different from $\theta_p = 1$ for a random walk. This indicates that $Z(t', \Delta t)$ is indeed long-range correlated in time.

For comparison, we have also performed the measurements with the data of the Shanghai Index from January 1998 to July 2003. The time interval between successive records is 5 minutes. As shown in Fig. 1, similar to the case of the German DAX, $P_-(t)$ obeys a power law and $P_+(t)$ decays faster. A quasi-periodic

dropping of $P_-(t)$ in early times is also observed, but the period now is less than 300 minutes, because the working day in China is about or less than five hours in those years. If one measures the the slope of the curve in the time interval $[500, 20000]$, the persistence exponent $\theta_p = 0.97(2)$ is obtained.

In general, the behavior of $P_+(t)$ is not expected to be universal, since it is not power-law-like, and it depends essentially on Δt in calculating $Z(t', \Delta t)$. In Fig. 1, $P_+(t)$ of the Shanghai Index decays slower than that of the German DAX, just because $\Delta t = 5\ mins$ for the Shanghai Index and $\Delta t = 1\ min$ for the German DAX. As Δt increases, the high-low asymmetry is continuously weakened. This tendency can be clearly seen in the analysis of the daily data in the next subsection.

Since $P_-(t)$ obeys a power-law, one may expect its behavior is universal. However, it is somewhat puzzling that the persistence exponent $\theta_p = 0.97(2)$ of the Shanghai Index is so close to $\theta_p = 1.0$, the value of a random walk. This point will further be investigated in the next subsection.

2.2 The daily data and minute-to-minute data

To push forward our investigation starting with the minute-to-minute data in the previous subsection, we now perform the measurements with the daily data of the German DAX from October 1959 to January 1997, and those of the Shanghai Index from December 1990 to December 2003. In Fig. 2, $P_+(t)$ and $P_-(t)$ of the daily data with $\Delta t = 1\ day$ are plotted on a log-log scale. In Fig. 2, we observe:

i) For both indies, compared with the case of the minute-to-minute data, it seems $P_+(t)$ comes closer to a power law, but still remains different from $P_-(t)$. This confirms that the behavior of $P_+(t)$ depends on Δt.

ii) For both indies, at least up to 100 day, one observes a power-law behavior of $P_-(t)$. The persistence exponent is estimate to be $\theta_p = 0.90(2)$ for the German DAX, and $\theta_p = 0.81(2)$ for the Shanghai Index.

The persistence exponent $\theta_p = 0.90(2)$ measured with the daily data of the

German DAX is consistent within the errors with $\theta_p = 0.88(2)$ with the minute-to-minute data. This indicates that the behavior of $P_{-}(t)$ is 'universal' for the German DAX from $\Delta t = 1$ *min* to 1 *day*. However, $\theta_p = 0.81(2)$ measured with the daily data of the Shanghai Index is very different from $\theta_p = 0.97(2)$ measured with the minute-to-minute data. To see this clearly, we rescale the time unit in Fig. 1 to a working day, and compare the curves with those in Fig. 2. This is shown in Fig. 3.

The difference between the minute-to-minute data and daily data of the Shanghai Index should not come from the disconnection between two working days, since this disconnection exists also for the German DAX. We conjecture that the minute-to-minute data of the Shanghai Index may be disturbed by many 'microscopic interactions' from the environment, and therefore, the dynamic behavior is different from that of the daily data. To confirm our conjecture, we perform additional calculations for the minute-to-minute data of the Shanghai Index with $\Delta t = 300$ *min*, which is about a working day. In other words, we uniformly select one datum from the records of every 300 minutes. Sometimes, we call this procedure 'renormalization'.

The persistence probabilities obtained with $Z(t', \Delta t = 300\ mins)$ of the Shanghai Index are displayed in Fig. 4, in comparison with those of the daily data. Interestingly, both the minute-to-minute data and daily data tend to exhibit similar dynamic behaviors and give a same persistence exponent. It seems that after we renormalize the time scale of the minute-to-minute data to $\Delta t = 300$ *mins*, the effects of the microscopic interactions from the environment are eliminated. In this sense, the behavior of $P_{-}(t)$ is also universal for the Shanghai Index.

Similar calculations with $\Delta t = 400$ *mins* have been also performed for the minute-to-minute data of the German DAX, and the results are in agreement with those of the daily data.

Finally, summarizing both measurements with the minute-to-minute data and daily data, the persistence exponent is $\theta_p = 0.89(2)$ for the German DAX and $\theta_p =$

0.80(2) for the Shanghai Index. The different values of the persistence exponent for both indices indicates that the financial markets in Germany and China may belong to different universality classes.

2.3 Modelling the real markets

In this subsection, we present numerical simulations of some microscopic models of the financial markets for comparison.

The herding model proposed by Eguiluz and Zimmermann [10] is simple and interesting, in which the clusters form themselves dynamically during the time evolution, but it does not provides a long-range temporal correlation. Therefore, a feedback interaction is introduced: the rate of transmission of information at time t' depends on the price change at time $t'-1$. Then, volatility clustering is generated [32, 33].

In Fig. 5, $P_+(t)$ and $P_-(t)$ of the interacting herding model are compared with those of the minute-to-minute data of the German DAX . After taking an average of the time series $y(t')$ generated by simulations over every 4 time steps, the interacting herding model reproduces nicely the minute-to-minute behavior of the German DAX for both $P_+(t)$ and $P_-(t)$.

In Fig. 6, we compare the persistence probabilities of the daily data of the German DAX, a random walk, a Minority Game with an inactive strategy [34, 20] and the interacting dynamic herding model. $P_+(t)$ and $P_-(t)$ of the random walk, and $P_+(t)$ (not in the figure) and $P_-(t)$ of the Minority Game all show a power-law behavior with a slope of 1.0, obviously different from those of the experimental data. The curve of $P_-(t)$ of the interacting herding model is consistent well with that of the experimental data.

We have also investigated the persistence probabilities of some other models, it seems not so easy to reproduce the behavior of the real markets. Even though the Minority Game with an inactive strategy produces most stylized facts of the

financial markets, including the power-law behavior of the auto-correlation function, it does not offer a correct behavior for the persistence probabilities. The interacting herding model does produce the high-low asymmetry of the minute-to-minute data of the German DAX, and also a nontrivial persistence exponent $\theta_p = 0.93(2)$. But the 'renormalization' does not yield the results of the daily data, i.e, it is not possible to produce the curves of $P_+(t)$ and $P_-(t)$ of the daily data of German DAX in Fig. 2 by selecting one datum from every some hundred time steps. (The curve of $P_-(t)$ in Fig. 6 is obtained only by rescaling the time unit in Fig. 5 to a working day, to be compared with that of the daily data of the German DAX.) The underlying reason may be simple: the interacting herding model generates only the temporal correlation for $|\Delta y(t') = y(t'+1) - y(t')|$, the sign of $\Delta y(t') = y(t'+1) - y(t')$ is randomly given, and therefore, selecting one datum of $y(t')$ from every some hundred time steps will lose most information of the time correlation.

3 General persistence probabilities

For further understanding the dynamic behavior of financial markets, we introduce more general persistence probability distributions. Assuming that z_0 is a real positive number, we define the generalized persistence probability $P_+(t, z_0)$ ($P_-(t, z_0)$) as the probability that $Z(t' + \tilde{t}, \Delta t)$ has never been down (up) to $Z(t', \Delta t) - z_0$ ($Z(t', \Delta t) + z_0$) in time t, i.e., $Z(t' + \tilde{t}, \Delta t) > Z(t', \Delta t) - z_0$ ($Z(t' + \tilde{t}, \Delta t) < Z(t', \Delta t) + z_0$) for all $\tilde{t} < t$. At $z_0 = 0$, $P_+(t, z_0)$ and $P_-(t, z_0)$ coincide with the persistence probabilities defined in the previous section.

For $P_-(t, z_0)$, we may write down a generalized dynamic scaling form

$$P_-(t, z_0) = t^{-\theta_p} F_-(t^{\alpha_-} z_0), \tag{2}$$

α_- is an exponent describing the scaling behavior of z_0. Obviously the power-law behavior in Eq. (1) is recovered at $z_0 = 0$. In this sense, $z_0 = 0$ is a fixed point in the dynamic system

Differentiation of Eq. (2) with respective to z_0 leads to

$$\partial_{z_0} P_-(t, z_0)|_{z_0=0} \sim t^{-\theta_p+\alpha_-}. \tag{3}$$

In Fig. 7, $\partial_{z_0}\ P_-(t, z_0)|_{z_0=0}$ has been displayed for the minute-to-minute data of both the German DAX and Shanghai Index. The curves decay roughly by a power law, and the corresponding exponent α_- is negative for both indices. This indicates that z_0 corresponds to an irrelevant parameter around the fixed point $z_0 = 0$. In other words, it induces only corrections to scaling. Corrections to scaling are usually not universal, and so is the exponent α_-.

In Fig. 8, $\partial_{z_0} P_-(t, z_0)$ has been plotted for different values of z_0 for the daily data of both the German DAX and Shanghai Index. For big values of z_0, the curves of both indices look approaching a power-law behavior with a positive slope about 0.3. This indicates that z_0 corresponds to a relevant parameter. As z_0 changes from 0 to a large value, it occurs a crossover behavior. Similar results are also obtained by analyzing the minute-to-minute data of both indices. At this point, besides the scales of z_0, we are not able to detect any significant difference between the German DAX and Shanghai Index within the accuracy of our data.

For $P_+(t, z_0)$, we do not observe any power-law behavior at $z_0 = 0$ for both indices, and the standard scaling form does not exist. To understand the effect of z_0, however, we may still suggest a 'quasi-scaling' form:

$$P_+(t, z_0) = f(t) F_+(g(t) z_0). \tag{4}$$

Its derivative leads to

$$\partial_{z_0} P_+(t, z_0)|_{z_0=0} \sim f(t) g(t). \tag{5}$$

In Fig. 7 and 8, $\partial_{z_0}\ P_+(t, z_0)|_{z_0=0}$ has been displayed for the minute-to-minute data and daily data of both indices. In all cases, it tends to a constant, i.e., $g(t) \sim 1/f(t)$, and $g(t)$ is an increasing function of t. In this sense, z_0 corresponds to a relevant parameter.

For comparison, we have also performed a similar analysis for the interacting herding model. In Fig. 7, $\partial_{z_0}\ P_-(t, z_0)|_{z_0=0}$ and $\partial_{z_0}\ P_+(t, z_0)|_{z_0=0}$ have been displayed by the cross lines. The model qualitatively reproduces the minute-to-minute behavior of the real markets.

In the case of $z_0 \neq 0$, a similar cross-over behavior as in Fig. 8 is also observed in the numerical simulations of the dynamic herding model. But the slope of the curve with a big z_0 is rather close to 1.0, different from 0.3 of the real market.

4 Conclusions

In conclusions, we have investigated the persistence probabilities $P_\pm(t, z_0)$ defined with the magnitude of the logarithm price change of the financial index, using the data of the German DAX and Shanghai Index. A power-law behavior is observed for $P_-(t, z_0 = 0)$ up to some months for both indices. The minute-to-minute data and daily data of the German DAX consistently yield a persistence exponent, $\theta_p = 0.89(2)$, while the minutely data and daily data of the Shanghai Index do not give a same persistence exponent. However, if we 'renormalize' the minutely data of the Shanghai Index to the time scale of a working day, i.e., calculate $Z(t', \Delta t)$ with $\Delta t = 300\ mins$, a 'universal' persistence exponent $\theta_p = 0.80(2)$ is obtained. These results indicate that both the German DAX and Shanghai Index are indeed long-range correlated in time, but they very probably belong to different universality classes.

$P_+(t, z_0 = 0)$ decays much faster than $P_-(t, z_0 = 0)$ and does not show a universal scaling behavior. This reflects a high-low asymmetry. In addition, we find that the parameter z_0 around $z_0 = 0$ corresponds to an irrelevant parameter for $P_-(t, z_0)$, while a relevant parameter for $P_+(t, z_0)$. The parameter z_0 tends to be relevant at the big values.

We have performed numerical simulations of an interacting herding model and a

Minority Game with an inactive strategy as well as some other models. In the case of z_0 around zero, the interacting herding model nicely reproduces the scaling behavior of the minute-to-minute data of the real markets, at least for the German DAX. It remains challenging to simulate the dynamics of the daily data and the behavior far away from the fixed point $z_0 = 0$. Further understanding of the Shanghai Index is also needed.

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References

[1] R. N. Mantegna and H. E. Stanley, Nature **376**, 46 (1995).

[2] S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner and Y. Dodge, Nature **381**, 767 (1996).

[3] R. N. Mantegna and H. E. Stanley, Nature **383**, 587 (1996).

[4] D. Sornette, A. Johansen and J-P. Bouchaud, J. Phys. I France **6**, 167 (1996).

[5] J. A. Feigenbaum and P. G. O. Freund, Int. J. Mod. Phys. **B10**, 3737 (1996).

[6] D. Stauffer and T. J. P. Penna, Physica **A256**, 284 (1998).

[7] L. Laloux, P. Cizeau, J-P. Bouchaud and M. Potters, Phys. Rev. Lett. **83**, 1467 (1999).

[8] V. Plerou, P. Gopikrishnan, B. Rosenow, L. A. N. Amaral and H. E. Stanley, Phys. Rev. Lett. **83**, 1471 (1999).

[9] T. Lux and M. Marchesi, Nature **397**, 498 (1999).

[10] V.M. Eguiluz and M.G. Zimmermann, Phys. Rev. Lett. **85**, 5659 (2000).

[11] H.E. Stanley, L.A.N. Amaral, X. Gabaix, P. Gopikrishnan and V. Plerou, Physica **A299**, 1 (2001).

[12] I. Giardina, J.P Bouchaud and M. Mézard, Physica **A299**, 28 (2001).

[13] P. Gopikrishnan, V. Plerou, L. A. N. Amaral, M. Meyer and H. E. Stanley, Phys. Rev. **E60**, 5305 (1999).

[14] Y. Liu, P. Gopikrishnan, P. Cizeau, M. Meyer, C-K. Peng and H. E. Stanley, Phys. Rev. **E60**, 1390 (1999).

[15] D. Challet and Y.C. Zhang, Physica **A246**, 407 (1997).

[16] D. Stauffer, P. M. C. de Oliveria and A. T. Bernardes, Int. J. Theor. Appl. Finance **2**, 83 (1999).

[17] D. Stauffer and N. Jan, Physica **A277**, 215 (2000).

[18] R. Cont and J-P. Bouchaud, Macroeconomic Dyn. **4**, 170 (2000).

[19] J.F. Muzy, J. Delour and E. Bacry, Eur. Phys. J. **B17**, 537 (2000).

[20] D. Challer, M. Marsili and Y.C. Zhang, Physica **A294**, 514 (2001).

[21] Y. Louzoun and S. Solomon, Physica **A302**, 220 (2001).

[22] A. Krawiecki, J.A. Holyst and D. Helbing, Phys. Rev. Lett. **89**, 158701 (2002).

[23] Z.F. Huang and S. Solomon, Physica **A306**, 412 (2002).

[24] S.N. Majumdar, A.J. Bray, S. Cornell and C. Sire, Phys. Rev. Lett. **77**, 3704 (1996).

[25] S. N. Majumdar, CURR SCI INDIA **77**, 370 (1999).

[26] B. Derrida, A.J. Bray and C. Godréche, J. Phys. **A27**, L357 (1994).

[27] S.N. Majumdar, C. Sire, A.J. Bray and S. Cornell, Phys. Rev. Lett. **77**, 2867 (1996).

[28] K. Oerding and F. van Wijland, J. Phys. **A31**, 7011 (1998).

[29] B. Zheng, Int. J. Mod. Phys. **B12**, 1419 (1998), review article.

[30] B. Zheng, Mod. Phys. Lett. **B16**, 775 (2002).

[31] F. Ren and B. Zheng, Phys. Lett. **A313**, 312 (2003).

[32] B. Zheng, F. Ren, S. Trimper and D.F. Zheng, Physica **A343**, 653 (2004).

[33] B. Zheng, T. Qiu and F. Ren, Phys. Rev. **E69**, 046115 (2004).

[34] P. Jefferies, M.L. Hart, P.M. Hui and N.F. Johnson, Int. J. Th and Appl. Fin **3**, 3 (2000).

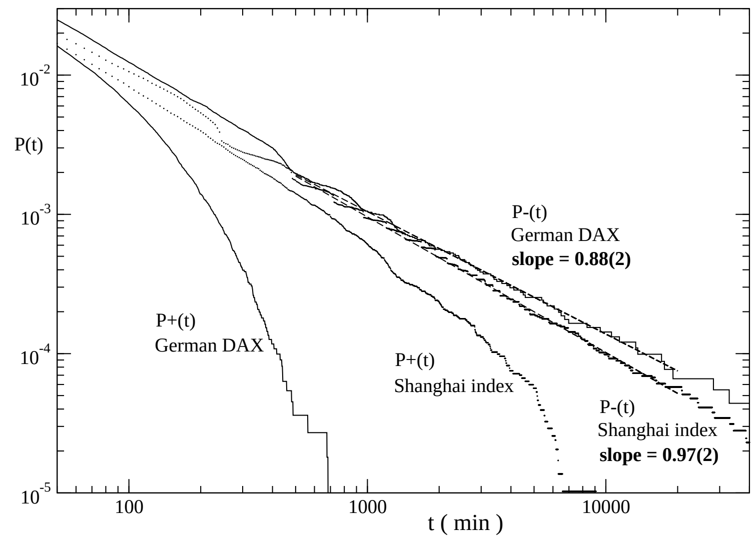

Figure 1: Persistence probabilities displayed in log-log scale. Curves of the German DAX (solid lines) are obtained with the minute-to-minute data and $\Delta t = 1$ *minute*, while those of the Shanghai Index (cross lines) are calculated with the data recorded every 5 minutes and $\Delta t = 5$ *minutes*. Dashed lines fitted to the curves show the power-law fits.

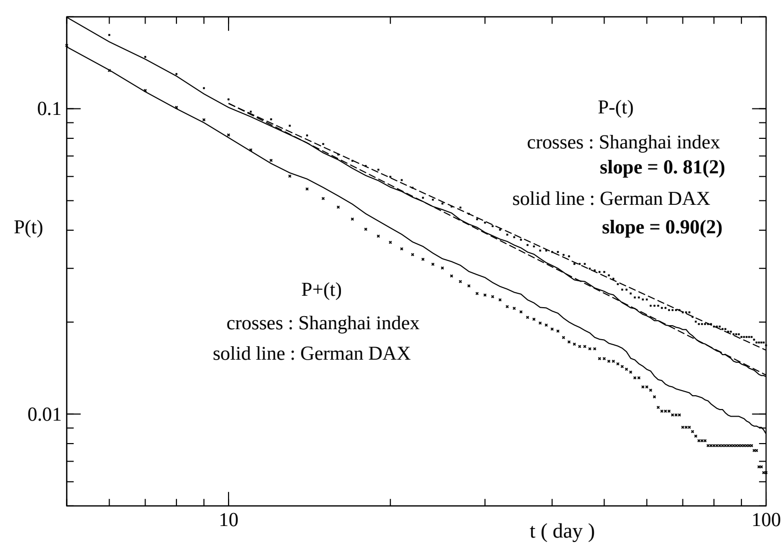

Figure 2: Persistence probabilities of the daily data in log-log scale. Curves are obtained with $\Delta t = 1$ *day*.

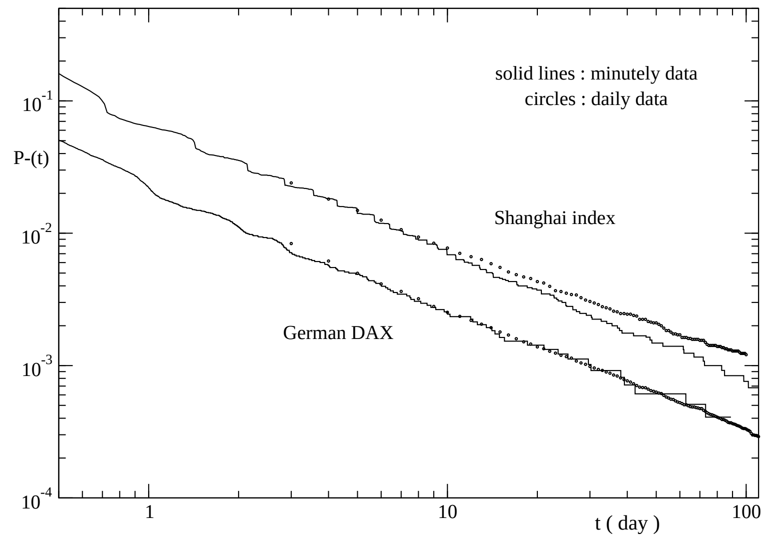

Figure 3: Comparison of the persistence probabilities measured with the daily data and minutely data. The curves are taken from Figs. 1 and 2. The time unit for the curves of the minutely data is 1 $day = 400\ mins$ for the German DAX and 1 $day = 300\ mins$ for the Shanghai Index.

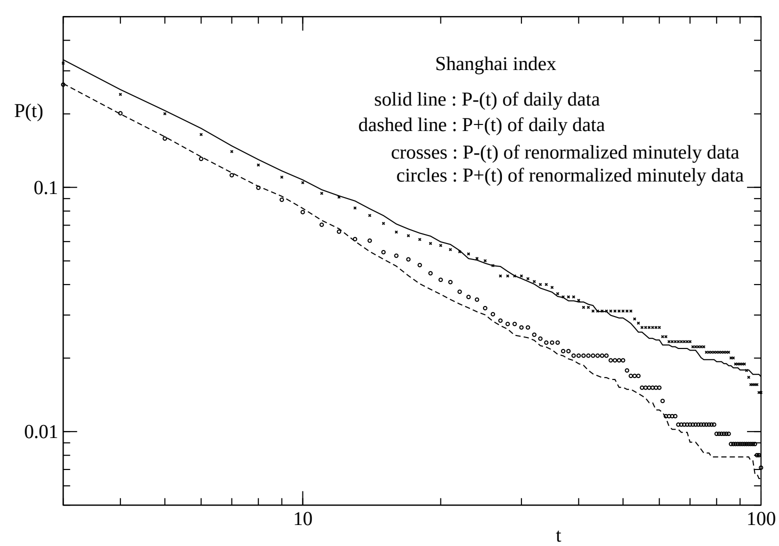

Figure 4: Persistence probabilities of the Shanghai Index in log-log scale. The curves of the minutely data are calculated with $\Delta t = 300\ mins$, i.e., with the data selected every 300 $mins$.

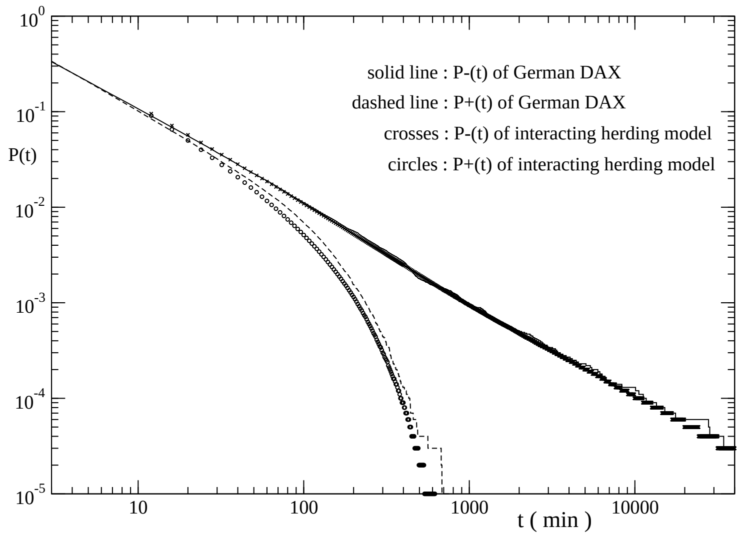

Figure 5: Persistence probabilities of the minutely data of the German DAX are compared with those of the interacting herding model.

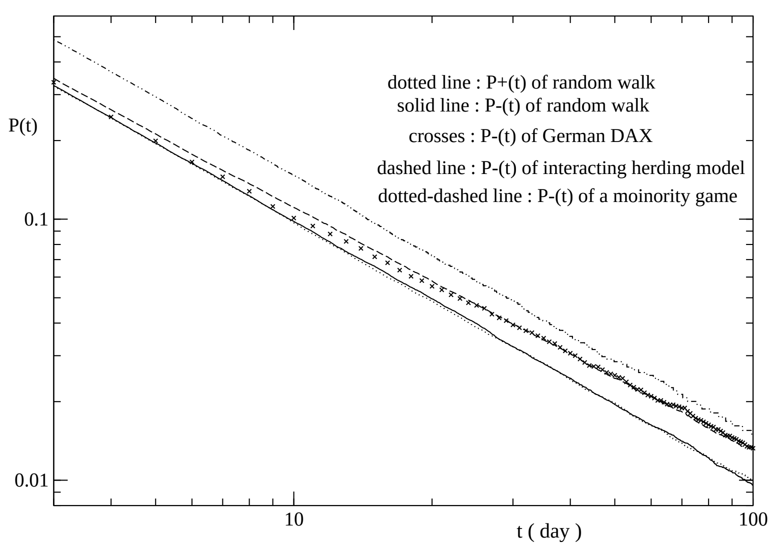

Figure 6: Persistence probabilities of the daily data of the German DAX are compared with those of the random walk, the Minority Game and the interacting herding model.

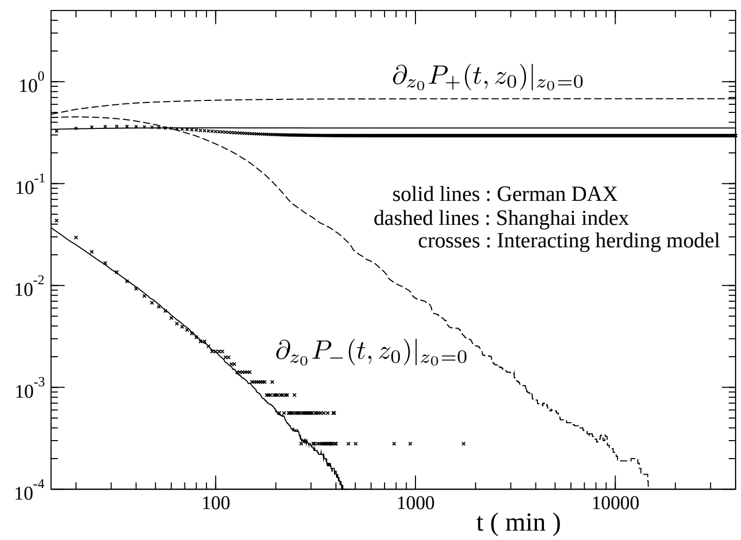

Figure 7: Derivatives of the persistence probabilities $P_\pm(t, z_0)|_{z_0=0}$ for the minutely data in log-log scale.

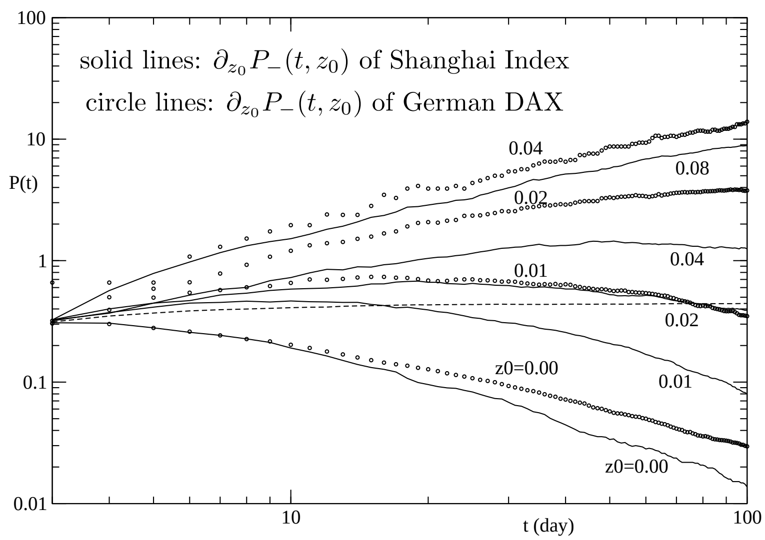

Figure 8: Derivatives of the persistence probabilities $P_\pm(t, z_0)$ for the daily data in log-log scale. The dashed line is $\partial_{z_0} P_+(t, z_0)|_{z_0=0}$ of the Shanghai Index.